Longitudinal Dispersion Coefficient in Straight Rivers

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Abstract: An analytical method is developed to determine the longitudinal dispersion coefficient in Fischer's triple integral expression for natural rivers. The method is based on the hydraulic geometry relationship for stable rivers and on the assumption that the uniform-flow formula is valid for local depth-averaged variables. For straight alluvial rivers, a new transverse profile equation for channel shape and local flow depth is derived and then the lateral distribution of the deviation of the local velocity from the cross-sectionally averaged value is determined. The suggested expression for the transverse mixing coefficient equation and the direct integration of Fischer's triple integral are employed to determine a new theoretical equation for the longitudinal dispersion coefficient. By comparing with 73 sets of field data and the equations proposed by other investigators, it is shown that the derived equation containing the improved transverse mixing coefficient predicts the longitudinal dispersion coefficient of natural rivers more accurately.

INTRODUCTION

The longitudinal dispersion of pollutants in rivers is important to practicing hydraulic and environmental engineers for designing outfalls or water intakes and for evaluating risks from accidental releases of hazardous contaminants. The ability of rivers or other surface water bodies to disperse added substances in longitudinal, lateral, and vertical directions (denoted, respectively, by x, y, and z) is measured by the dispersion coefficients k_x, k_y, and k_z, respectively. The longitudinal disperson coefficient was first introduced by Taylor (1953, 1954) as a measurement of the 1D dispersion process described by the classical advection-dispersion equation (Fukuoka and Sayre 1973; McQuivey and Keefer 1976; Fischer et al. 1979; Sukhodolov et al. 1997)

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = K_x\frac{\partial^2 C}{\partial x^2} \tag{1}$$

in which C = cross-sectionally averaged concentration; U = mean longitudinal velocity; t = time; and x = longitudinal coordinate oriented in the direction of mean flow. Eq. (1) holds only after the so-called initial period or after the Fickian period is reached. It indicates that the transport process and hence the fate of pollutants in surface water bodies depend to a large extent on the longitudinal dispersion coefficient k_x. For this reason, the longitudinal dispersion coefficient has been investigated extensively since the original work of Taylor (1953, 1954), who showed that, at some distance downstream from an injection in a pipe flow, a tracer reaches a balance between advection and diffusion.

Elder (1959) extended Taylor's result from pipes to open channels and derived an equation to compute the longitudinal dispersion coefficient based on the equilibrium between longitudinal velocity shear and vertical turbulent diffusion. Fischer et al. (1979) found that the transverse profile of longitudinal velocity is 100 or more times as important in producing longitudinal dispersion as the vertical profile in natural rivers. Using an analysis similar to that of Taylor and Elder and taking into account the balance between longitudinal advective mass transport and transverse diffusive mass transport, Fischer et al. (1979) developed an integral expression of the following form for the longitudinal dispersion coefficient for rivers:

$$K_x = -\frac{1}{A}\int_0^B hu'\int_0^y \frac{1}{\varepsilon_t h}\int_0^y hu'\,dy\,dy\,dy \tag{2}$$

where A = cross-sectional area; y = coordinate in the lateral direction; $h = h(y)$ = local flow depth; u' = deviation of local depth mean velocity from the cross-sectional mean velocity; B = channel width; and $\varepsilon_t = \varepsilon_t(y)$ = local transverse mixing coefficient. Eq. (2) has been employed as the basis of various empirical methods determining the longitudinal dispersion coefficient. However, there is a misconception concerning parameter ε_t.

It is widely adopted that ε_t represents the transverse turbulent diffusion coefficient (Koussis and Rodriguez-Mirasol 1998; Seo and Cheong 1998; Piasecki and Katopodes 1999) for natural rivers. Fischer et al. (1979) took parameter $\varepsilon_t = 0.15Hu_*$ as the transverse turbulent diffusion coefficient only for the idealized rivers with a uniform, straight, infinitely wide channel of constant depth, as there is no transverse dispersion for such a case (here H is the cross-sectionally averaged depth). Noting that in natural rivers the effect of transverse dispersion on the longitudinal dispersion coefficient K_x is larger than that of transverse turbulent diffusion, Rutherford (1994) took the parameter ε_t as the cross-sectional mean value of the transverse dispersion coefficient instead of the transverse turbulent diffusion coefficient. Actually, lateral mixing in natural rivers is a complex process involving both shear dispersion and turbulent diffusion. It is difficult, if not impossible, to distinguish the effect of dispersion from that of turbulent diffusion in reality. However, it is convenient to express ε_t as a sum of the two different kinds of mixing process in theory; i.e., $\varepsilon_t = K_y + E_y$, where K_y refers to the transverse dispersion coefficient and E_y refers to the transverse turbulent diffusion coefficient.

The fundamental difficulty in determining K_x from (2) is the lack of the knowledge of transverse profiles of both velocity $u(y)$ and depth $h(y)$. Hence, numerous investigations into estimation of K_x have been empirical. By qualitative analysis and simplification of (2) and use of $\varepsilon_t = 0.6Hu_*$, Fischer et al. (1979) obtained an approximate formula for K_x, expressed in the form

$$K_x = 0.011\left(\frac{U}{u_*}\right)^2\left(\frac{B}{H}\right)^2 Hu_* \tag{3}$$

The advantage of (3) is that the dimensionless dispersion coefficient $K_x/(Hu_*)$ can be estimated from readily available bulk hydraulic variables, width-to-depth ratio B/H, and friction term

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U/u_*. Such a feature is instructive indeed. Applying the one-step Huber method, Seo and Cheong (1998) gave the following regression equation and stated that the equation was superior to existing evaluations for predicting the dispersion coefficient of natural streams:

$$\frac{K_x}{Hu_*} = 5.915 \left(\frac{B}{H}\right)^{0.62} \left(\frac{U}{u_*}\right)^{1.428} \tag{4}$$

The objective of this study is (1) to determine the lateral distribution of river flow depth $h(y)$, which is the key impediment to a theoretical prediction of longitudinal dispersion coefficient K_x by means of Eq. (2); (2) to determine the transverse mixing coefficient ε_t; and (3) to derive an equation, based on Eq. (2), for predicting the longitudinal dispersion coefficient, which is theoretically reasonable and is as accurate as or more accurate than existing equations.

TRANSVERSE DISTRIBUTION OF LOCAL FLOW DEPTH

The transverse distribution of local flow depth depends on the channel shape for a natural river. Owing to its importance, the cross-sectional shape of stable channels has long been the subject of numerous investigations (ASCE 1998). The channel shapes proposed by different investigators can be classified into three types: cosine channel, exponential channel, and parabolic channel. However, these channel shapes are only applicable to canals or to the bank regions of straight rivers. To predict the cross-sectional shape of natural alluvial rivers, the channels are usually generalized with a flat-bed region and two curving bank regions (Vigilar and Diplas 1997). The width of the flat-bed region is determined numerically. It means that no available channel shape equation can be directly used to simulate the cross-sectional channel shape of natural rivers. To establish a simple equation describing the river channel shape, it is assumed that the river channel is straight, its cross section is symmetrical about its center and is constant along the river, as shown in Fig. 1.

The cross-sectional channel shape of an alluvial river is governed by its hydraulic geometry, referring to the interrelationship among water discharge, channel width, flow depth, velocity, and so forth. The hydraulic geometry of rivers is distinguished between at-a-station hydraulic geometry and downstream hydraulic geometry. At a river cross section, the water surface width B and mean flow depth H vary with discharge. Formulas for these relationships were given as power functions of the discharge by Chang (1988), Richards (1982), and Chien et al. (1987), among others

$$B = aQ^{\delta} \tag{5}$$

$$H = dQ^{\theta} \tag{6}$$

where a, d, δ, and θ = numerical constants. The average values of the exponents δ and θ have been obtained by Chien et al. (1987) for 374 river cross sections representing a large variety of rivers from all over the world. These average values are $\delta = 0.14$ and $\theta = 0.43$. The channel shape with $\delta = 0.14$ and $\theta = 0.43$ corresponds to the rivers with the highest occurring frequency in nature (Park 1977) and thus is the most stable channel shape (Deng and Singh 1999). It is logical to define a channel shape parameter, $\beta = \theta/\delta$. The value $\beta \approx 3.07$ for stable rivers in dynamic equilibrium. However, a majority of natural rivers are not in the dynamic equilibrium state and therefore their channel shape parameter β should be a variable rather than a constant value. Eqs. (5) and (6) lead to the following at-a-station hydraulic relation between width and mean depth with a constant e:

$$B = eH^{1/\beta} \tag{7}$$

The generalized parabolic curve is $h = H_c - py^q$, which with $y = 0$ at the centerline and $y = b$ at the shore means that $H_c = pb^q$. The mean depth in a cross section is then $H_c q/(q + 1)$ or $[pq/(q + 1)]b^q$. To satisfy the relationship of (7), q must be equal to β. With β as the only parameter, the dimensionless depth variation in a cross section is

$$\frac{h(y)}{H_c} = 1 - \left(\frac{y}{b}\right)^{\beta} \tag{8}$$

When $\beta = 2$, (8) turns to the typical parabolic channel shape equation of stable canals. In general, the value of parameter β is >2 and should closely depend on the width-to-depth ratio B/H for natural rivers. The value of $\beta = 2$ is justified by Cao and Knight (1997) for threshold alluvial channels with $B/H = 8$ or $\ln(B/H) \approx 2.08$. Moreover, using the model on plan geometry of meandering rivers proposed by Chang (1988), it is found that $B/H = 21$ or $\ln(B/H) \approx 3.04$ for stable straight rivers with an arc angle close to zero. As mentioned earlier, the straight stable rivers have a channel shape parameter $\beta = \theta/\delta \approx 3.07$. It is interesting to find $\beta = \ln(B/H)$ when $B/H = 8$ or 21. On these grounds, it is then inferred that a functional relationship exists between the channel shape parameter β and the channel width-to-depth ratio B/H

$$\beta = \ln\left(\frac{B}{H}\right) \tag{9}$$

Eq. (8) in conjunction with (9) is a useful mathematical means describing the cross-sectional channel shape of natural rivers because of its adaptability to variable channel shapes. For instance, (8) represents a triangular shape for $\beta = 1$, parabolic shape for $\beta = 2$, approximate natural channel shape with a flat-bed region and two curving bank regions for $\beta > 2$ (say, $\beta = 5$), and rectangular shape for $\beta = \infty$, as shown in Fig. 2, where $H_c = b = 1$. Fig. 2 demonstrates that (8) is able to reflect

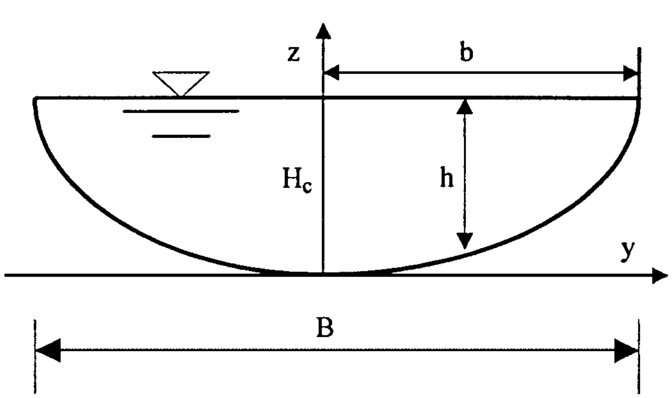

FIG. 1. Coordinate System and Generalized Channel Profile

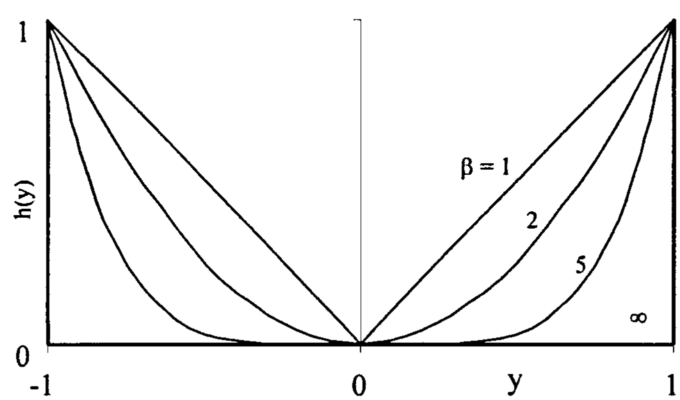

FIG. 2. Variation of Channel Shape with β

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different cross-sectional shapes of channels with size ranging from small canals to large rivers. The cross-sectional channel shape is a significant factor in determining the actual magnitude of the longitudinal dispersion coefficient of streams (Sooky 1969). Therefore, (8) in combination with (9) is essential to the determination of the lateral velocity distribution and thus of the longitudinal dispersion coefficient.

From (8), the cross-sectionally averaged flow depth is obtained

$$H = H_c \int_0^1 (1 - \xi^\beta)\, d_\xi = \frac{\beta}{\beta + 1} H_c \qquad (10)$$

where a dimensionless lateral coordinate $\xi = y/b$ is introduced. The cross-sectionally averaged velocity is determined from the Manning formula

$$U = \frac{\sqrt{S}}{n} R^{2/3} \qquad (11a)$$

where the hydraulic radius can be determined analytically if the parameter $\beta = 2$. When a channel is very wide, the hydraulic radius approaches the mean depth in the cross section. For $\beta = 2$, when $B/H = 8$, the hydraulic radius is 0.8 of the mean depth and is close to unity when the ratio increases. This means that, for not very narrow channels, the section-averaged velocity can be expressed by (11*b*) with U_c as the mean velocity at the centerline

$$U = \frac{\sqrt{S}}{n} H_c^{2/3} \left(\frac{\beta}{\beta + 1}\right)^{2/3} = U_c \left(\frac{\beta}{\beta + 1}\right)^{2/3} \qquad (11b)$$

where S = channel slope; n = Manning roughness coefficient; and H_c = flow depth at the channel center. With the channel shape assumed in Fig. 1, the river is characterized by the uniformity of longitudinal slope S and surface roughness n across the channel boundary. Moreover, careful experiments indicate that the sides of a channel have practically no influence on the velocity distribution in the central region when the width is >10 times the depth of flow (Chow 1959). Most natural straight rivers or river reaches approximately meet the above conditions. It means that the uniform flow condition is roughly valid for individual verticals in most parts of the cross section of the straight natural streams and rivers with width-to-depth ratio >10. Consequently, it is reasonable to assume that the Manning equation still holds for the local depth-averaged velocity for the straight channels with width-to-depth ratio >10; i.e.,

$$u(y) = \alpha \frac{\sqrt{S}}{n} h(y)^{2/3} \qquad (12)$$

in which α = revision coefficient accounting for the difference between flow depth and hydraulic radius satisfying the following constraint:

$$\int_0^B h'_u \, dy = 0 \qquad (13)$$

where the deviation of the velocity $u(y)$ from the cross-sectional mean u' is expressed

$$u' = u(y) - U = \alpha \frac{\sqrt{S}}{n} h^{2/3} - U$$

or

$$u' = \left[\alpha \left(\frac{h}{H}\right)^{2/3} - 1\right] U \qquad (14)$$

Substituting (8) and (10) into (14) and using $\xi = y/b$, one obtains

$$u' = \left[\alpha(1 - \xi^\beta)^{2/3} \left(\frac{\beta + 1}{\beta}\right)^{2/3} - 1\right] U \qquad (15)$$

Eq. (15) describes the lateral distribution of the deviation of the velocity $u(y)$ from the cross-sectional mean.

DETERMINATION OF TRANSVERSE MIXING COEFFICIENT

Eq. (2) contains three unknown variables, namely, local flow depth $h(y)$, velocity deviation u', and lateral mixing coefficient ε_t. The variables h and u' can be determined from (8) and (15), respectively. Up to now, a theoretical expression of ε_t is not available. This expression should include K_y and E_y, which should also be determined.

A large number of experiments have been conducted to determine the transverse turbulent diffusion coefficient E_y. Webel and Schatzmann (1984) carried out a systematic experimental study and found that E_y/u_*H has a constant value of 0.13, which approaches $E_y/u_*H = 0.17$, measured in a straight section of the river Rhine near Karlsruhe. Based on the result of a total of 75 separate experiments in straight channels, Fischer et al. (1979) suggested that $E_y = 0.15u_*H$. Rutherford (1994) summarized 139 sets of experimental data and found that E_y/u_*H lies in the range of 0.10–0.26. The authors' calculations for the 139 sets of data showed that 138 sets of data resulted in the values of E_y/u_*H ranging from 0.10 to 0.26. One set of data gave an extremely high value of $E_y/u_*H = 0.847$; thus, this data set was not used. Using 138 sets of experimental data collected by Rutherford from different investigators, the authors found a constant transverse turbulent diffusion coefficient

$$\frac{E_y}{Hu_*} = 0.145 \qquad (16)$$

The result is consistent with $E_y = 0.15u_*H$, suggested by Fischer et al. (1979). The result can also be expressed in the form

$$\frac{E_y}{Bu_*} = \frac{0.145}{B/H} \qquad (17)$$

The 138 sets of data give a regression equation with a correlation coefficient of $R^2 = 0.9248$, as shown in Fig. 3. The difference between the dimensionless values predicted by the regression equation and (17) is <0.0005 when $B/H > 10$. It is therefore sufficiently accurate to use (16) or (17) because of their simplicity, compared to the regression equation.

To obtain a laterally distributed turbulent diffusion coefficient, it is assumed that (16) is still valid for local flow depth and u_* denotes the cross-sectionally averaged shear velocity

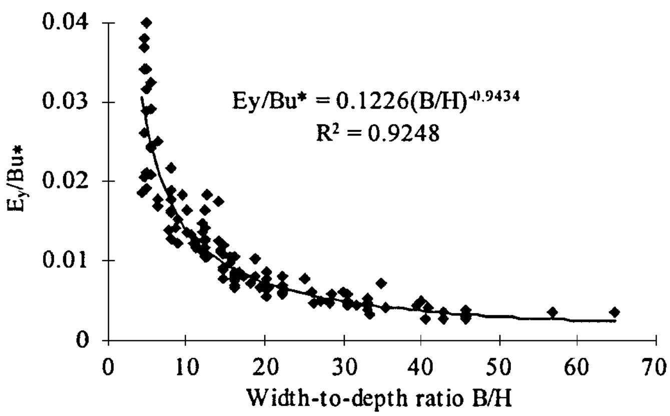

FIG. 3. Relationship between E_y/u_*B and B/H

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$$E_y = 0.145 u_* h(y) \quad (18)$$

Several attempts have been made to establish a relationship between transverse dispersion coefficient K_y and bulk river channel parameters. To estimate the transverse dispersion coefficient K_y in natural meandering rivers, Fischer, Yotsukura, and Sayre developed their equations (Rutherford 1994). These equations lead to the result that there is no transverse dispersion in straight rivers. However, field measurements indicate that transverse dispersion depends on secondary currents rather than on turbulence generated by bed friction, whereas the secondary currents exist even in straight natural rivers (Nezu et al. 1993). Therefore, transverse dispersion exists in all natural rivers, including straight ones.

For large rivers, the following transverse dispersion equation has been proposed (Smeithlov 1990):

$$\frac{K_y}{Hu_*} = \left(\frac{1}{3{,}520}\right)\left(\frac{U}{u_*}\right)\left(\frac{B}{H}\right)^{1.38} \quad (19)$$

Eq. (19) was established based on data measured on 11 rivers in the United States. The rivers have different characteristics. Some of the rivers are shallow and wide, and some are deep and narrow. Eq. (19) is also assumed to be valid for local flow depth. That is, the cross-sectionally averaged flow depth H can be replaced by the variable local flow depth $h(y)$ when (19) is applied to (2)

$$\frac{K_y}{hu_*} = \left(\frac{1}{3{,}520}\right)\left(\frac{U}{u_*}\right)\left(\frac{B}{H}\right)^{1.38} \quad (20)$$

Eqs. (18) and (20) lead to the transverse mixing coefficient

$$\varepsilon_t = \left[0.145 + \left(\frac{1}{3{,}520}\right)\left(\frac{U}{u_*}\right)\left(\frac{B}{H}\right)^{1.38}\right] u_* h \quad (21)$$

Let

$$\varepsilon_{t0} = 0.145 + \left(\frac{1}{3{,}520}\right)\left(\frac{U}{u_*}\right)\left(\frac{B}{H}\right)^{1.38} \quad (22)$$

Then

$$\varepsilon_t = \varepsilon_{t0} u_* h \quad (23)$$

DERIVATION OF LONGITUDINAL DISPERSION COEFFICIENT

Using (8), (15), and (23), it is plausible to predict the longitudinal dispersion coefficient K_x theoretically by a direct integration of (2). Substituting (8), (15), and (23) into (2) and accounting for the symmetry of the generalized river channel shown in Fig. 1, (2) can be written

$$K_x = -\frac{1}{A}\int_{+b}^{-b} hu' \int_b^y \frac{1}{\varepsilon_t h}\int_b^y hu'\, dy\, dy\, dy$$

$$= -\frac{2}{BH}\int_1^0 hu' \int_1^\xi \frac{b^3}{\varepsilon_{t0} u_* h^2}\int_1^\xi hu'\, d_\xi\, d_\xi\, d_\xi$$

$$= -\frac{2}{BH}\left(\frac{B}{2}\right)^3 \int_1^0 (1-\xi^\beta) H_c \left[\alpha(1-\xi^\beta)^{2/3}\left(\frac{\beta+1}{\beta}\right)^{2/3} - 1\right]$$

$$\cdot U \int_1^\xi \frac{1}{\xi_{t0} u_* (1-\xi^\beta)^2 H_c^2} \cdot \int_1^\xi (1-\xi^\beta) H_c$$

$$\cdot \left[\alpha(1-\xi^\beta)^{2/3}\left(\frac{\beta+1}{\beta}\right)^{2/3} - 1\right] U d_\xi\, d_\xi\, d_\xi = -\frac{2}{8\varepsilon_{t0}}\left(\frac{B^2}{H}\right)$$

$$\cdot \left(\frac{U^2}{u_*}\right)\int_1^0 (1-\xi^\beta)\left[\alpha(1-\xi^\beta)^{2/3}\left(\frac{\beta+1}{\beta}\right)^{2/3} - 1\right]$$

$$\cdot \int_1^\xi \frac{1}{(1-\xi^\beta)^2}\int_1^\xi (1-\xi^\beta) \cdot \left[\alpha(1-\xi^\beta)^{2/3}\right.$$

$$\left.\cdot \left(\frac{\beta+1}{\beta}\right)^{2/3} - 1\right] d_\xi\, d_\xi\, d_\xi$$

Let

$$I = \int_{+1}^{-1} (1-\xi^\beta)\left[\alpha(1-\xi^\beta)^{2/3}\left(\frac{\beta+1}{\beta}\right)^{2/3} - 1\right]\int_1^\xi \frac{1}{(1-\xi^\beta)^2}$$

$$\cdot \int_1^\xi (1-\xi^\beta)\left[\alpha(1-\xi^\beta)^{2/3}\left(\frac{\beta+1}{\beta}\right)^{2/3} - 1\right] d_\xi\, d_\xi\, d_\xi \quad (24)$$

Then, the dimensionless longitudinal dispersion coefficient can be expressed

$$\frac{K_x}{B_{u*}} = -\frac{1}{8\varepsilon_{t0}}\left(\frac{B}{H}\right)\left(\frac{U}{u_*}\right)^2 I(\beta) \quad (25)$$

As mentioned earlier, both the local flow depth $h(y)$ and the deviation $u'(y)$ of the local velocity from the cross-sectional mean velocity are defined based on the straight symmetrical channel and the uniform flow. However, natural rivers involve many kinds of nonuniformities, such as dead zones, bends, and islands (Sooky 1969; Rutherford 1994). There even exists secondary flow in straight natural rivers (Nezu et al. 1993). These nonuniformities of channel geometry and flow affect the theoretical definitions of $h(y)$ and $u'(y)$ and thus the dispersion coefficient K_x. It is therefore necessary to introduce a revision parameter in (25) to account for the various nonuniformities involved in both flow and geometrical characteristics of natural rivers. Some experimental results provide quantitative information about the influence of nonuniformities.

Fischer (1967) conducted a series of dispersion experiments in laboratory channels with smooth and rough banks. The two sets of experiments were purposefully made under conditions as nearly identical as possible, except for the bank roughness. The experimental results show that the longitudinal dispersion coefficient in the channel with rock sides was about 15 (14.7) times that in the smooth channel. It is apparent that the channel, assumed for the derivation of (25) and shown in Fig. 1, corresponds to the smooth one; whereas most real channels possess rough sides due to the bank vegetation, groins, irregular bank alignment formed by natural erosion of flow, and other nonuniformities mentioned earlier. To make (25) applicable to natural rivers and streams, it is amended by multiplying the right-hand side by a comprehensive revision constant ψ, leading to the following equation:

$$\frac{K_x}{Bu_*} = -\frac{\psi}{8\varepsilon_{t0}}\left(\frac{B}{H}\right)\left(\frac{U}{u_*}\right)^2 I(\beta) \quad (26a)$$

or

$$\frac{K_x}{Hu_*} = -\frac{\psi}{8\varepsilon_{t0}}\left(\frac{B}{H}\right)^2\left(\frac{U}{u_*}\right)^2 I(\beta) \quad (26b)$$

where the comprehensive revision constant ψ can be taken as a constant of 15 based on Fischer's experimental result. The value $\psi = 15$ can be confirmed by using field data.

The value of $I(\beta)$ is the dimensionless triple integral, which quantifies the velocity variation over the channel cross section and is defined in (24). The value of $I(\beta)$ is only dependent on the channel shape parameter β or width-to-depth ratio B/H. For a given river, $I(\beta)$ can be solved by numerical integration

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TABLE 1. Numerical Integration Results for Different Channel Width-to-Depth Ratios

B/H	β	α	I
10	2.3026	0.9058	−0.004786
20	2.9957	0.9230	−0.003601
50	3.9120	0.9380	−0.002623
100	4.6052	0.9460	−0.002126
150	5.0106	0.9500	−0.001851
200	5.2983	0.9525	−0.001775

of (24). To avoid the complicated triple integration, $I(\beta)$ is computed for different channel width-to-depth ratios B/H and the calculated results are given in Table 1. The six sets of data listed in Table 1 produce a correlation curve between the channel width-to-depth ratio B/H and the triple integration value $I(\beta)$, as shown in Fig. 4. The curve can be described simply and accurately by the following regression equation:

$$I = -\frac{0.01}{(B/H)^{1/3}} \tag{27}$$

Substitution of (27) into (25) and (26) yields the dimensionless longitudinal dispersion coefficient equations, respectively

$$\frac{K_x}{Bu_*} = \frac{0.01}{8\varepsilon_{t0}}\left(\frac{B}{H}\right)^{2/3}\left(\frac{U}{u_*}\right)^2 \tag{28a}$$

or

$$\frac{K_x}{Hu_*} = \frac{0.01}{8\varepsilon_{t0}}\left(\frac{B}{H}\right)^{5/3}\left(\frac{U}{u_*}\right)^2 \tag{28b}$$

$$\frac{K_x}{Bu_*} = \frac{0.01\psi}{8\varepsilon_{t0}}\left(\frac{B}{H}\right)^{2/3}\left(\frac{U}{u_*}\right)^2 \tag{29a}$$

or

$$\frac{K_x}{Hu_*} = \frac{0.01\psi}{8\varepsilon_{t0}}\left(\frac{B}{H}\right)^{5/3}\left(\frac{U}{u_*}\right)^2 \tag{29b}$$

The process of numerical integration of (24) is demonstrated in the Appendix for $B/H = 20$ and $\beta = \ln(B/H) \approx 3$. The symbols I_1 and I_2 in the Appendix are defined respectively as follows:

$$I_1 = \int_1^{\xi} (1 - \xi^{\beta})\left[\alpha(1 - \xi^{\beta})^{2.3}\left(\frac{\beta + 1}{\beta}\right)^{2/3} - 1\right] d_{\xi}$$

$$I_2 = \int_1^{\xi} \frac{1}{(1 - \xi^{\beta})^2}\int_1^{\xi} (1 - \xi^{\beta})\left[\alpha(1 - \xi^{\beta})^{2/3}\left(\frac{\beta + 1}{\beta}\right)^{2/3} - 1\right] d_{\xi}\, d_{\xi}$$

To avoid an irrational prediction of flow depth, the dimensionless transverse distance ξ should be taken as positive (i.e., $\xi = |\xi|$ if $\xi < 0$), as shown in column 2 in the Appendix. The numerical computation uses 20 subareas, as listed in Table 4. In column 5, I_1 represents the cumulative dimensionless discharge from one bank of the channel to the other. Each value in column 6 is obtained by taking the value at the end of the previous subarea and adding the mean of the cumulative dimensionless discharge multiplied by the width ($\Delta\xi = 0.1$) of the subarea and divided by the value of $(1 - \xi^{\beta})^2$. Each value in column 7 is obtained by taking the value at the end of the previous subarea and adding the mean of column 6 in the subarea multiplied by the dimensionless discharge in the subarea and by its width ($\Delta\xi = 0.1$). The last figure in column 7 is the value of the triple integral in (24). Following the same procedure, the integral value of (24) can be calculated for any given β. Owing to the symmetry of the generalized channel shape, the figures in column 6 also show the symmetry when ξ varies from 0.1 to 1.0 and from −0.1 to −1.0. Such a symmetry makes the last figure −0.0017986 in the line $\xi = 0.0$ equal half of the last figure −0.0036013 in the last line $\xi = -1.0$. Consequently, the numerical integration is conducted only for $\xi = 0.0$ to 1.0 for other width-to-depth ratios.

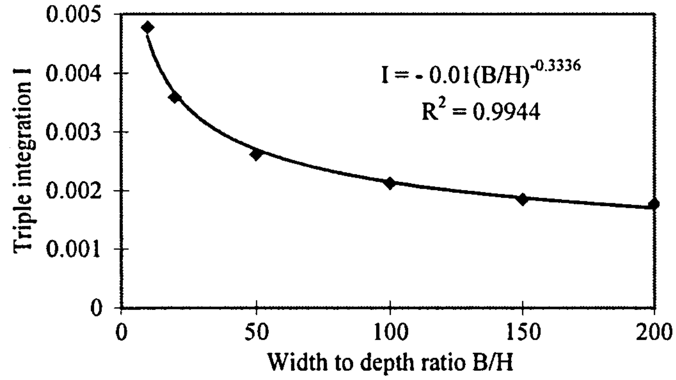

FIG. 4. Variation of Triple Integration $I(\beta)$ with Channel Width-to-Depth Ratio B/H

VERIFICATION OF NEW LONGITUDINAL DISPERSION COEFFICIENT EQUATION

The accuracy of the newly established equation for the longitudinal dispersion coefficient and the suitability of the revision constant $\psi = 15$ were evaluated using 73 sets of data measured on 29 rivers in the United States (Table 2). The following two criteria were taken into consideration in the data selection process: (1) the data measured on the U.S. rivers should be preferable because the transverse dispersion coefficient [(19)] was established based on the data from the rivers in the United States rather than from canals—thus, the selected data did not include the data from canals; (2) to facilitate a comparison with other equations, the same data adopted for comparable equations should be used. According to these criteria, among the 59 sets of data employed by Seo and Cheong (1998), 58 data sets (1–58 in Table 2) were adopted. Furthermore, 15 sets of data on rivers from the United States collected by Rutherford (1994) were also selected (59–73).

A comparison between the measured and computed values of the dispersion coefficient further demonstrates that it is reasonable to define the revision constant $\psi = 15$. Substituting $\psi = 15$ into (29) yields a new equation for the longitudinal dispersion coefficient for natural rivers

$$\frac{K_x}{Hu_*} = \frac{0.15}{8\varepsilon_{t0}}\left(\frac{B}{H}\right)^{5/3}\left(\frac{U}{u_*}\right)^2 \tag{30}$$

where ε_{t0} can be computed from (22). Eq. (30) stems from the direct integration of (2) and is thus theoretically based. Moreover, (30) not only includes the conventional parameters, channel width-to-depth ratio B/H, and friction term U/u_* but also involves the effect of transverse mixing ε_{t0}. This distinguishing feature of (30) is that it clarifies its dispersion mechanism. In addition, (30) is conducive to further improvement if a more accurate transverse dispersion equation is found.

As mentioned earlier, (4) is regarded as being superior to existing equations (Seo and Cheong 1998) in explaining dispersion characteristics of natural streams. To compare with other equations, (4) is therefore employed to do the same calculation of the longitudinal dispersion coefficient. The computed results are listed in the last two columns of Table 2. These results indicate that the new (30) provides predictions closer to the measured values of the longitudinal dispersion coefficient than does (4). Among 58 data sets collected by Seo and Cheong (1998), 32 predictions by (30) and 30 by (4) fall within the range of $0.5 < D_{\mathrm{prediction}}/D_{\mathrm{measurement}} < 2$; 34 data sets

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TABLE 2. Comparison of Measured and Predicted Longitudinal Dispersion Coefficients

Number	River	Width B (m)	Depth H (m)	Velocity U (m/s)	Shear velocity u_* (m/s)	Dispersion Coefficient K_x (m^2/s) Measured value	Predicted by Eq. (30)	Predicted by Eq. (4)
1	Antietam Creek, Md.	12.80	0.30	0.42	0.057	17.50	17.55	17.96
2		24.08	0.98	0.59	0.098	101.50	47.25	53.68
3		11.89	0.66	0.43	0.085	20.90	14.97	20.17
4		21.03	0.48	0.62	0.069	25.90	44.37	46.93
5	Monocacy River, Md.	48.70	0.55	0.26	0.052	37.80	28.21	27.14
6		92.96	0.71	0.16	0.046	41.40	25.79	23.53
7		51.21	0.65	0.62	0.044	29.60	85.52	110.87
8		97.54	1.15	0.32	0.058	119.80	72.18	70.94
9		40.54	0.41	0.23	0.040	66.50	20.08	20.35
10	Conococheague Creek, Md.	42.21	0.69	0.23	0.064	40.80	22.92	20.80
11		49.68	0.41	0.15	0.081	29.30	11.74	9.27
12		42.98	1.13	0.63	0.081	53.30	93.05	96.69
13	Chattahoochee River, Ga.	75.59	1.95	0.74	0.138	88.90	168.61	169.12
14		91.90	2.44	0.52	0.094	166.90	146.56	148.04
15	Salt Creek, Nebr.	32.00	0.50	0.24	0.038	52.20	20.71	20.58
16	Difficult Run, Va.	14.48	0.31	0.25	0.062	1.90	9.45	9.02
17	Bear Creek, Colo.	13.72	0.85	1.29	0.553	2.90	28.13	52.28
18	Little Pincy Creek, Md.	15.85	0.22	0.39	0.053	7.10	16.22	16.91
19	Bayou Anacoco, La.	17.53	0.45	0.32	0.024	5.80	21.82	25.00
20	Comite River, La.	15.70	0.23	0.36	0.039	69.00	15.77	17.39
21	Bayou Bartholomew, La.	33.38	1.40	0.20	0.031	54.70	23.00	26.28
22	Amite River, La.	21.34	0.52	0.54	0.027	501.40	46.68	59.89
23	Tickfau River, La.	14.94	0.59	0.27	0.080	10.30	9.66	11.76
24	Tangipahoa River, La.	31.39	0.81	0.48	0.072	45.10	49.06	50.01
25		29.87	0.40	0.34	0.020	44.00	28.67	39.22
26	Red River, La.	253.59	1.62	0.61	0.032	143.80	270.87	473.56
27		161.54	3.96	0.29	0.060	130.50	134.42	132.88
28		152.40	3.66	0.45	0.057	227.60	230.01	238.12
29		155.14	1.74	0.47	0.036	177.70	181.13	235.11
30	Sabine River, La.	116.43	1.65	0.58	0.054	131.30	188.76	218.91
31		160.32	2.32	1.06	0.054	308.90	508.34	718.79
32	Sabine River, Tex.	14.17	0.50	0.13	0.037	12.80	4.59	5.23
33		12.19	0.51	0.23	0.030	14.70	10.49	11.87
34		21.34	0.93	0.36	0.035	24.20	32.73	37.46
35	Mississippi River, La.	711.20	19.94	0.56	0.041	237.20	1,617.50	1,854.49
36	Mississippi River, Mo.	533.40	4.94	1.05	0.069	457.70	1,244.96	1,792.98
37		537.38	8.90	1.51	0.097	374.10	2,578.86	3,271.43
38	Wind/Bighorn River, Wyo.	44.20	1.37	0.99	0.142	184.60	150.87	158.73
39		85.34	2.38	1.74	0.153	464.60	577.23	638.00
40	Copper Creep, Va.	16.66	0.49	0.20	0.080	16.84	6.91	7.64
41	Clinch River, Va.	48.46	1.16	0.21	0.069	14.76	23.77	23.47
42	Copper Creek, Va.	18.29	0.38	0.15	0.116	20.71	3.96	4.16
43	Powell River, Tenn.	36.78	0.87	0.13	0.054	15.50	9.89	9.93
44	Clinch River, Va.	28.65	0.61	0.35	0.069	10.70	28.39	27.52
45	Copper River, Va.	19.61	0.84	0.49	0.101	20.82	28.39	33.75
46	Clinch River, Va.	57.91	2.45	0.75	0.104	40.49	158.03	179.90
47		53.24	2.41	0.66	0.107	36.93	118.28	139.68
48	Copper Creek, Va.	16.76	0.47	0.24	0.080	24.62	9.31	9.79
49	Missouri River, Iowa	180.59	3.28	1.62	0.078	1,486.45	1,008.41	1,382.00
50	Bayou Anacoco, La.	25.91	0.94	0.34	0.067	32.52	26.82	29.61
51		36.58	0.91	0.40	0.067	39.48	45.49	45.69
52	Nooksack River, Wash.	64.01	0.76	0.67	0.268	34.84	82.63	69.65
53	Wind/Bighorn River, Wyo.	59.44	1.10	0.88	0.119	41.81	156.59	159.96
54		68.58	2.16	1.55	0.168	162.58	405.53	437.37
55	John Day River, Oreg.	24.99	0.58	1.01	0.140	13.94	81.63	83.23
56		34.14	2.47	0.82	0.180	65.03	71.16	116.81
57	Yadkin River, N.C.	70.10	2.35	0.43	0.101	111.48	83.82	91.21
58		71.63	3.84	0.76	0.128	260.13	177.13	277.02
59	Minnesota River	80.00	2.74	0.034	0.0024	22.3	12.05	13.88
60		80.00	2.74	0.14	0.0097	34.9	49.85	57.62
61	Amite River	37.00	0.81	0.29	0.070	23.2	28.42	27.29
62		42.00	0.80	0.42	0.069	30.2	50.96	50.18
63	White River	67.00	0.55	0.35	0.044	30.2	46.40	54.32
64	Nooksack River	86.00	2.93	1.20	0.53	153.0	195.54	239.79
65	Susquehanna River	203.00	1.35	0.39	0.065	92.9	134.88	150.05
66	Bayou Anacoco	20.00	0.42	0.29	0.045	13.9	17.59	17.54
67	Muddy River	13.00	0.81	0.37	0.081	13.9	12.80	18.98
68	Muddy River	20.00	1.20	0.45	0.099	32.5	24.10	34.94
69	Comite River	13.00	0.26	0.31	0.044	7.0	12.30	12.43
70		16.00	0.43	0.37	0.056	13.9	19.41	19.88
71	Missouri river	183.00	2.33	0.89	0.066	465.0	437.93	558.77
72		201.00	3.56	1.28	0.084	837.0	847.27	1,054.37
73	Missouri River	197.00	3.11	1.53	0.078	892.0	950.80	1,317.33

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favor (30) over (4), which was derived from the data sets, whereas 24 sets of the data favor (4). Among the total of 73 sets of data listed in Table 2, 44 measured dispersion coefficients are closer to those predicted by (30) than to those predicted by (4). In other words, the new (30) gives closer predictions in 60.3% of cases, as opposed to 39.7% by the empirical formula of Seo and Cheong. In the extreme case, the predicted dispersion coefficient of (30) deviates from the measured value by a factor of 10 (No. 17 and No. 22) and (4) overestimates the dispersion coefficient by a maximum factor of 18 (No. 17). It follows from the above comparison that the proposed equation is capable of providing a superior prediction of the longitudinal dispersion coefficient for natural rivers.

DISCUSSION OF ANALYSIS

Sensitivity Analysis

A sensitivity and error analysis of the new longitudinal dispersion coefficient equation is conducted for mean values of input and output variables in (30) and on the assumption that the errors in each input variable are independent. The 73 sets of data in Table 2 give the average values of the channel width, flow depth, velocity, shear velocity, and dispersion coefficient as B = 83.46 m, H = 1.72 m, U = 0.556 m/s, u_* = 0.087 m/s, and K_x = 132.00 m^2/s, respectively.

If the error ΔK in output longitudinal dispersion coefficient K_x is defined as the difference between values of K_x predicted for inputs $X + \Delta X$ and X, then the error can be estimated using a truncated Taylor series or the absolute sensitivity $A_s = \partial K/\partial X$ (ASCE 1996); i.e., $\Delta K = K(X + \Delta X) - K(X) \approx (\partial K/\partial X)\cdot\Delta X$, where ΔX is the error in model input X denoting the variables B, H, U, or u_*. The error could also be expressed in a relative form: $\Delta K/K$. The error ΔK of the above equation is essentially the deviation sensitivity with ΔX being the error. The relative sensitivity R_s can be expressed as $R_s = (\partial K/\partial X)\cdot(X/K)$ (ASCE 1996). Assuming that each predictor variable is incremented by a constant percentage of 10%, then the errors ΔK in dispersion coefficients are computed, as shown in Table 3. Table 3 indicates that the velocity U is the most sensitive variable among the four input variables; thus, the same change of 10% in U causes the greatest variation in the dispersion coefficient K_x. The channel width B is next in importance, followed by depth H and shear velocity u_*. The relative sensitivity of U is about twice that of B, which is roughly twice that of H as well. Therefore, the prediction accuracy of (30) depends heavily on the value of velocity U and its distribution. This means that accurate measurements of flow velocity U and channel width B can significantly improve predictions by (30).

Influence of Flow and Channel Geometry Change on Dispersion Coefficient

Table 2 illustrates the variability of the dispersion coefficient in different streams. Actually, K_x changes even in the same stream with flow and hence water level. Eq. (30) can be recast by using the Manning formula and shear velocity expression

$$K_x = \frac{0.15}{8\varepsilon_{t0}}\left(\frac{1}{n^2}\sqrt{\frac{S}{g}}\right)B^{5/3}H^{1/6} \tag{31a}$$

$$\varepsilon_{t0} = 0.145 + \frac{1}{3{,}520}\left(\frac{H^{1/6}}{n\sqrt{g}}\right)\left(\frac{B}{H}\right)^{1.38} \tag{31b}$$

TABLE 3. Sensitivity and Error Analysis of New Dispersion Coefficient Eq. (30)

X	A_s	R_s	ΔX	ΔK	Relative error
B	1.110	0.702	8.346 m	9.268	0.070
H	27.306	0.356	0.172 m	4.697	0.036
U	319.672	1.347	0.056 m/s	17.774	0.135
u_*	−439.036	−0.289	0.009 m/s	3.820	0.029

Eq. (31) indicates that the longitudinal dispersion coefficient K_x increases with flow depth H provided that the water level is maintained in the main channel or the flow discharge is less than the bank-full one. Otherwise, Manning's roughness coefficient n may increase significantly once the discharge exceeds the bank-full one, causing the decrease of K_x. Such a behavior of K_x with flow is consistent with experimental results (Guymer 1998). The experimental results of both Guymer (1998) and Fischer (1967) have illustrated that compound or more natural cross-sectional geometry channel increases greatly the value of the longitudinal dispersion coefficient. Further investigation is needed to quantify the effects of channel and flow nonuniformities on the longitudinal dispersion coefficient K_x.

Limitations of Analysis

The basic assumptions of this analysis limit the application of the new longitudinal dispersion coefficient equation to straight uniform rivers in theory. The differences between observed and predicted dispersion coefficients are mainly attributed to the effects of dead zones, bends, secondary currents, and other irregular features that are not explicitly involved in (30). The introduction of the comprehensive revision constant ψ reflects the effects of these irregularities in real rivers to some extent. Note that ψ = 15 is defined for natural rivers with moderate irregularity. The value of ψ should be equal to unity in theory and be slightly greater than unity in practice for smooth channels such as canals with lined banks [for instance, the predicted longitudinal dispersion coefficient k_x = 6.2 m^2/s by (28) and the measured one k_x = 9.6 m^2/s for the Coachella Canal with B = 24.40 m, H = 1.56 m, U = 0.71 m/s, and u_* = 0.044 m/s (Fischer 1968)]. The values of the predicted and measured dispersion coefficients result in ψ = 1.5 close to the theoretic value of ψ = 1. It means that the predicted dispersion coefficient by (28) is reasonable at least for the assumed channel in Fig. 1. Moreover, the majority of streams are uniform enough for an approximate analysis (Fischer et al. 1979). Therefore, the new equation can be used practically for natural rivers that approximately satisfy 1D flow conditions with relative high accuracy, as demonstrated in Table 2. Furthermore, Fischer's triple integral expression [(2)] of the longitudinal dispersion coefficient, on which (30) is based, is valid only after the initial convective-dominated period or after a distance L downstream from the source where the balance between advection and diffusion is reached. The distance L satisfies the condition $L > 0.4UB^2/\varepsilon_t$ according to Fischer et al. (1979). Finally, (2) was derived for river channels with large width-to-depth ratios >6 (Fischer 1967) and the Manning equation can be employed to individual verticals in the river channels with B/H > 8 to 10. Therefore, the use of (30) is suggested for natural rivers and streams having width-to-depth ratio >10.

CONCLUSIONS

Using the hydraulic geometry relationship of stable rivers, a channel shape equation or a transverse profile equation of

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local flow depth is derived; thereby, the lateral distribution of the deviation of local depth mean velocity from the cross-sectional average value is determined for straight alluvial rivers. Using the suggested equation for the transverse mixing coefficient and the direct integration of Fischer's triple integral, a new equation for the longitudinal dispersion coefficient for natural rivers is derived by taking into account the irregularity of natural rivers. By comparing with 73 sets of field data and the equations proposed by other investigators, it is found that the new (30) has the least error in predicting the longitudinal dispersion coefficients for natural rivers. More than 64% of the predictions by the new equation fall within the range of $0.5 < K_{\text{prediction}}/K_{\text{measurement}} < 2$. Moreover, as compared with existing equations, the new equation is theoretically based, more accurate, and clarifies its dispersion mechanism. The distinct feature of the suggested equation lies in its incorporation of the transverse mixing coefficient. The new equation can be further improved if a more accurate transverse dispersion equation is found. The future investigation into longitudinal dispersion should highlight the determination of an accurate transverse dispersion coefficient equation and the influence of channel irregularity.

APPENDIX

TABLE 4. Numerical Integration of Eq. (24) for $\beta = 3$

ξ (1)	$\bar{\xi}$ of subarea (2)	Width $\Delta\xi$ (3)	$1 - \bar{\xi}^{\beta}$ (4)	I_1 (5)	I_2 (6)	$I(\beta)$ (7)
1.0	—	—	—	0	0	0
0.9	0.95	0.1	0.1426	−0.00991	−0.02436	0.000121
0.8	0.85	0.1	0.3859	−0.02563	−0.03629	0.000597
0.7	0.75	0.1	0.5781	−0.03858	−0.04590	0.001130
0.6	0.65	0.1	0.7254	−0.04564	−0.05390	0.001482
0.5	0.55	0.1	0.8336	−0.04644	−0.06053	0.001528
0.4	0.45	0.1	0.9089	−0.04197	−0.06588	0.001245
0.3	0.35	0.1	0.9571	−0.03375	−0.07001	0.000687
0.2	0.25	0.1	0.9844	−0.02327	−0.07295	−0.000062
0.1	0.15	0.1	0.9966	−0.01175	−0.07471	−0.000913
0.0	0.05	0.1	0.9999	0.00006	−0.07530	−0.001799
−0.1	0.15	0.1	0.9966	0.01181	−0.07471	−0.002684
−0.2	0.25	0.1	0.9844	0.02333	−0.07294	−0.003535
−0.3	0.35	0.1	0.9571	0.03381	−0.06999	−0.004284
−0.4	0.45	0.1	0.9089	0.04204	−0.06585	−0.004842
−0.5	0.55	0.1	0.8336	0.04651	−0.06049	−0.005124
−0.6	0.65	0.1	0.7254	0.04570	−0.05386	−0.005079
−0.7	0.75	0.1	0.5781	0.03865	−0.04584	−0.004727
−0.8	0.85	0.1	0.3859	0.02569	−0.03622	−0.004195
−0.9	0.95	0.1	0.1426	0.00997	−0.02424	−0.003720
−1.0	—	—	—	0.00007	0.00027	−0.003601

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NOTATION

The following symbols are used in this paper:

A = cross-sectional area of river channel (m^2);
A_s = absolute sensitivity;
a = numerical constant;
B = surface width of river channel (m);
b = $B/2$ (m);
C = cross-sectional average concentration (mg/L);
d = numerical constant;
E = turbulent diffusion coefficient (m^2/s);
H = cross-sectional average flow depth (m);
H_c = flow depth at channel center (m);
h = local flow depth (m);
I = value of dimensionless integral;
K = dispersion coefficient (m^2/s);
n = Manning roughness coefficient;
p = numerical constant;
q = numerical constant;
R_s = relative sensitivity;
S = channel slope;
t = time;
U = cross-sectional average velocity (m/s);
u = longitudinal velocity (m/s);
u' = deviation of local depth mean velocity from cross-sectional mean (m/s);
u_* = shear velocity (m/s);

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X = variable;
x = longitudinal coordinate;
y = lateral coordinate;
z = vertical coordinate;
α = revision coefficient;
β = channel shape parameter;
ΔK = error in dispersion coefficient;
ΔX = error in variable;
δ = numerical constant;
ε_t = transverse mixing coefficient (m^2/s);
θ = numerical constant;
ξ = dimensionless transverse distance y/b;
σ = numerical constant; and
ψ = revision constant.

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